THE WAY TO BEGIN SOLID GEOMETRY.

By Howard F. Hart.

The methods in solid geometry which I am now using in our high school have proved to be such an advance over former methods that I have called this paper, which describes those methods, by the presumptuous title above.

It has always seemed to me that the aims in solid geometry should be to continue those of plane geometry together with the visualization of the figures as they in reality exist in three dimensional space. I believe that readymade solutions for definitions, propositions, and exercises should not be given in advance to the students but should be the summaries of class discussions on those points. In the beginning of things it must have been that the figures were studied first and the theorems derived from them. Therefore the figures should precede. Furthermore, it should be directly or tacitly assumed that the figures studied exist. This is as logical and better than the present practice in which the existence of certain things is discussed at great length (*e. g.*, a perpendicular to a plane) while others just as vital and necessary (*e. g.*, the intersection of two planes or the intersection of a spherical surface by a plane in a closed line) are assumed. However, such propositions on existence as *e. g.*, the common perpendicular between two non-coplanar lines and the number of ways in which regular plane polygons can be used to form the faces of polyhedral angles should be discussed and demonstrated as they are different in kind.

So after a review of the logical relations of the converse, negative, and negative-converse of a proposition; the locus idea and what has to be proved in order to prove a locus; the loci of plane geometry; the definitions of solid, surface, plane surface (two definitions, for both are necessary) and straight line; and a discussion of what classes of plane geometry propositions are applicable to figures lying in more than one plane; the first

figure "two straight lines in space" is discussed. The class readily finds the three possibilities.

Of course in this early work, and constantly throughout, we have to be alert to use whatever material is at hand to help the student build up his space concepts. The class room is a never-failing source of help. The lines of the ceiling, side walls, and floor give all that is needed in many configurations. Strings from the gas jet to a desk are very helpful. Whatever models are used are such that they can be built up before the class. In this connection I find Hanstein's goniostat (Chicago Math. Supply House) to be very helpful and can commend it.

To return to our first figure: Two straight lines in space. As given above, there are three possibilities: (1) and (2) in the same plane, hence parallel or intersecting; (3) not in the same plane. I have found it very easy to deduce the fact that each of the first two possibilities determines a plane while the third makes it impossible for the lines to be either parallel or intersecting.

Then we study "Two Planes in Space." They may or may not have common points. The students naturally apply the right name to the second possibility and thus parallel planes have become in the right way (Dewey says in "How We Think" that anything learned without the use of thinking, *i. e.*, induction, is not educative) a part of the student's notions and the definition is a mere incident. A deeper questioning will develop the fact that the intersection of two planes must be defined as the locus of their common points. At this point I tell the class that since they seem to be unable to think of two planes as having only one common point, we will assume that they have two such points. From that it is but a step to see that all points on the line determined by these two points are common, that any common point is on the line, and that the line is therefore by definition the intersection of the planes. This is one new locus. Also we see what hereafter we must know about a line to call it the intersection of two planes.

For the third figure we consider the relative positions of a "Line and a Plane." The line may lie in the plane, be parallel to it, or intersect it. We find that apart from the definition (a plane is such that a straight line joining, etc.) a line lies in a plane if it is parallel to a line in the plane and has one

point in the plane but is parallel to the plane if parallel to the line and has no point in the plane. I find that the students in considering the third possibility without any questioning naturally subdivide it and name the cases. Then we hunt for the conditions which a line must satisfy to be perpendicular to a plane. I find that at this time the students need to have this question asked of them, "What would be the condition of a line which was known to be *not* perpendicular to one line in the plane through its foot." They see that the line *could not be* perpendicular to the plane ("It must slant" as one boy put it), and hence get the definition. Set to work to see how many lines it must be perpendicular to be perpendicular to all they soon find the answer. I reserve the discussion of the line oblique to a plane until after we have investigated the projection of a line on a plane.

The fourth configuration is that of "Three planes in space." The three planes may obviously intersect in three lines, in two lines, in one line, or not at all. If they intersect in three lines the lines the lines of intersection may either be concurrent or parallel as two of them are concurrent or parallel. The second of these alternatives is very important in later study of parallels. If the planes intersect in two lines then two of the planes do not intersect and are therefore parallel and the lines are parallel by definition. The third possibility gives us the arrangement called "Pencil of Planes," a useful idea. In the last the planes are all parallel to each other.

For the fifth subject we study the "Lines perpendicular to one line at the same point." Determining our planes carefully we find that (1) the given line and any one of the other lines are coplanar, (2) any three of the latter are coplanar, and that therefore they are all coplanar. The converse has already been studied in a line perpendicular to a plane; hence we can take this as a new locus at once.

These first five studies in the relative positions of lines and planes in space are doubtless enough of the beginning to indicate the aims and methods of the plan. It will be seen that we begin at the *beginning;* that thinking has been a maximum, telling a minimum; that the study of the figures produces theorems whose proof is but the orderly arrangement of the discussion.

I assume that it will be understood that many a class period is given over to formal recitations in which the orderliness of the work is carefully criticized, that many exercises are given in further relations of the figures and later, particularly, in mensuration. Finally let me say that success in solid geometry depends upon insight into the figures; that I know of no way to give a boy such an insight except to develop it.

And it can be developed.

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VICE.

The shadow of idleness.—Tennyson.

The daughter of pleasure.—Bushnell.

The tax-gatherer of the community.—Anon.

The greatest of all Jacobins; the arch-leveller.—Hare.

A burdensome fellow-traveller, a chargeable table-companion, and a troublesome bed-fellow.—Plutarch.

Such a self-sufficient worker of infelicity, that it has no need either of instruments or servants.—Plutarch.